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ERGODIC CHANNEL CAPACITY AS MIMO PERFORMANCE MEASURE

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Abstract: **This paper provide an overview of computing channel capacity for evaluating the performance of MIMO (Multiple input multiple output) wireless communication system. It primarily provide analysis and comparison of ergodic capacity assuming with independent and identically distributed (i. i. d.) environment between antenna pairs and Rayleigh flat fading channel. Moreover, a computer simulation with MATLAB is implemented. Correlation of real world wireless channel may result in substantially degrades the performance of MIMO architecture. Apart from this, effect of correlation coefficient and efficiency imbalance on ergodic capacity is also presented which is helpful for an antenna engineer whose objective is to attain the optimum antenna design.**

Keywords: **MIMO, Rayleigh flat fading, signal to noise, Shannon capacity, correlation coefficient, efficiency imbalance.**

I. INTRODUCTION

In research of wireless communications, multiple input multiple outputs (MIMO) are recent adoption [2]. MIMO System equipped with multiple antennas at both side of the link has recently drawn considerable attention in response to the increasing requirements on data rate and quality radio link. A number of channel models exist in MIMO wireless communication system. In deterministic channel, channel capacity is the maximum mutual information between input and output of the system. But in random channels, there is lot of channel capacity formulas. Channel capacity of MIMO architecture in independent Rayleigh channels scales linearly increases with number of antennas.

II. ERGODIC CAPACITY

The channel gain process is ergodic, i.e., the time average is equal to ensemble average. The ergodic capacity of a MIMO channel is the ensemble average of the information rate over the distribution of the elements of the channel matrix H. In the channels with fading, the notion of capacity is not convenient to illustrate the radio channel. When a deep fade occurs, no data can be transmitted. According to the definition, capacity of such a channel is equal to zero. Thus, instead of the capacity, two other notions are generally used. They are outage capacity and ergodic capacity.

In the channels where the fading is rapid, the expectation value of the capacity is usually calculated. Again, the channel can be in a deep fade, but these periods are short and the loss of data can be compensated by the suitable joint coding and interleaving. This expectation value is called ergodic capacity.

III. ERGODIC CAPACITY MATRIC

Consider an M×M MIMO channel, the instantaneous channel capacity with no channel information at the transmitter (i.e., identical transmit power distribution) can be expressed as [7]

$$C = \log_2\left(I_M + \frac{\rho_T}{M} HH^H\right) \tag{1}$$

Where the SNR is defined as $\rho_T = \frac{P_T}{\sigma_n^2}$. P_T denotes the transmit power and σ_n^2 is the noise power at receiver. Since the interest here is in antenna design, the propagation environment of independent and identically distributed Rayleigh fading channel H_w is assumed. MIMO channel is given by

$$H = R^{1/2} H_W \tag{2}$$

Where R is receive correlation matrix that completely describes the effect of antennas on channel.

$$R = \Lambda^{1/2}\,\overline{R}\,\Lambda^{1/2} \tag{3}$$

Where $\bar{R}$ is normalized correlation matrix whose diagonal elements are 1. And diagonal matrix given by

$$\Lambda = Diag[\eta_1, \eta_2, \eta_3, \ldots\ldots \eta_M] \tag{4}$$

Where η_i is the total efficiency of the i[th] antenna port. The instantaneous capacity (1) is given by [7]

$$C = C_0 + \log_2 det(R) \quad (5)$$

Where Co denotes the capacity of ideal i.i.d. Rayleigh channel at high SNR

$$C_0 = \log_2 det\left(\frac{\rho_T}{M} H_W H_W^H\right) \quad (6)$$

Ideal antennas have their efficiency are 100% and absolutely orthogonal to one another in radiation pattern (either in space and/or polarization). Since $\log_2 \det(\mathbf{\Lambda}) \leq 0$ and $\log_2 \det(\bar{\mathbf{R}}) \leq 0$ see also [6], non-ideal antenna effects will result in a constant degradation in the channel capacity over SNR relative to that of i.i d. channel.

In order to translate this capacity gap into a power related measure, we can apply the following equality [1]:

$$det(R) = det\left(det(R)^{1/M} I_M\right) \quad (7)$$

to (5), which can then be rewritten as

$$C \approx \log_2 det\left(\frac{\rho_T}{M} det(R)^{1/M} H_W H_W^H\right) \quad (8)$$

Comparing (8) to (6), we conclude that at high SNRs, the capacity C in (5) with non-ideal antennas is equivalent to that of ideal antennas in i.i.d. channel with the SNR

$$\rho_0 = \rho_T \, det(R)^{1/M} \quad (9)$$

In this context, the multiplexing efficiency is defined as

$$\eta_{mux} = \frac{\rho_0}{\rho_T} \leq 1 \quad (10)$$

or equivalently

$$\eta_{mux}(dB) = \rho_0 - \rho_T \leq 0 \quad (11)$$

which measures the loss of efficiency in SNR (or power, assuming the noise power σ_n^2 is the same) when using a real multiple antenna prototype in an i.i.d. channel (with SNR ρ_T) to achieve the same capacity as that of an ideal array in the same i.i.d. channel (with SNR ρ_0).

For high SNRs, η_{mux} is readily obtained from (9), i.e.,

$$\tilde{\eta}_{mux} = \lim_{\rho_T \to \infty} \eta_{mux} = det(\bar{R})^{\frac{1}{M}} \quad (12)$$

Substituting (3) into (12), we can rewrite

$$\tilde{\eta}_{mux} = det(\Lambda\bar{R})^{\frac{1}{M}} = \left(\prod_{k=1}^{M} \eta_k\right)^{1/M} det(\bar{R})^{1/M} \quad (13)$$

which shows that the multiplexing efficiency is determined by the product of the geometric mean (or the arithmetic mean in dB scale) of the antenna efficiencies and a correlation induced term $det(\bar{R})^{\frac{1}{M}}$. The geometric mean term is intuitive in that the overall efficiency should come in between the efficiencies of the constituent antennas. For the correlation induced term, its impact can be understood in that, as the correlation among the ports increases, the condition number of $\bar{R}$ increases. This in turn decreases both $det(\bar{R})$ and $(\bar{R})^{\frac{1}{M}}$. In other words, a higher ρ_T is needed in order for its capacity to match that of the i.i.d. case with SNR ρ_0.

In general, the definition (10) is still valid, even when the high SNR assumption is not satisfied. However, the resulting expression for η_{mux} is more involved and is a function ρ_T of (or equivalently ρ_0). In other words, the constant capacity gap seen in (5) may not apply.

The procedure for deriving the exact η_{mux} for a given instantaneous realization of H_W is given as follows.

• Equate the i.i.d. capacity C_{iid} (of SNR ρ_0) with (1), and by the property of determinant

$$C_{iid} = \log_2 det\left(I_M + \frac{\rho_0}{M} H_W H_W^H\right)$$

$$C = \log_2 det\left(I_M + \frac{\rho_T}{M} R H_W H_W^H\right) \quad (14)$$

• Since the function $\log_2(\,.\,)$ is monotonic

$$det\left(I_M + \frac{\rho_0}{M} H_W H_W^H\right)$$
$$= det\left(I_M + \frac{\rho_T}{M} R H_W H_W^H\right) \quad (15)$$

• Introduce (10) in (15) and solve for η_{mux} numerically.

Whereas $\tilde{\eta}_{mux}$ in (12) does not depend on either the channel realization or the exact SNR η_{mux}, is influenced by both factors. In practice, MIMO performance is typically characterized by ergodic capacity, which is calculated from a large number of Monte Carlo realizations of the channel matrix H. Thus, it is more appropriate to derive η_{mux} based on the ergodic capacity. This is achieved by first taking the expectation on both sides of (14)

$$E\{C_{iid}\} = E\{C\} \quad (16)$$

However, there is no known exact closed-form solution for (16). As an alternative, for a given SNR ρ_T, the ergodic capacity for the non-ideal antennas is determined from Monte Carlo simulations, and the required SNR ρ_0 for the ideal antennas to offer the identical ergodic capacity can then be obtained by a parametric search (e.g., by decreasing ρ_0 progressively from a starting guess of $\rho_0 = \rho_T$). Hence, η_{mux} can be calculated numerically from the given ρ_T and the corresponding solution ρ_0 using (10).

One way to get around this cumbersome approach is to take the upper bound from Jensen's inequality on both sides of (16), which after some manipulations yields

$$(1+\rho_0)^M = \det\left(I_M + \rho_T R\right) \quad (17)$$

Then, substituting into (17) and solving for, we obtain

$$\eta_{mux} = \frac{\left(det(I_M+\rho_T R)^{1/M} - 1\right)}{\rho_T} \quad (18)$$

which can be shown to converge to (12) in the limit of high SNRs. However, it is noted that the closed-form solution (18) is obtained using the upper bounds for Jensen's inequality, which is in fact a loose bound, as can be seen in [7]. Moreover, the calculation involves taking the roots of a polynomial in and does not readily offer constructive insights into the impact of non-ideal multiple antennas, as is possible with (13). In addition, since it is deterministic, it can be shown that the high SNR solution (12) is equally valid for ergodic capacity, in spite of having been derived based on instantaneous capacity.

IV. CASE STUDY

In 2×2 MIMO, there are two receiving antenna and two transmit antenna. If efficiency and normalized correlation matrix is given by [1]

$$\Lambda = \begin{matrix} \eta_1 & 0 \\ 0 & \eta_2 \end{matrix}$$

$$\bar{R} = \begin{matrix} 1 & r \\ r^* & 1 \end{matrix}$$

So resulting from the above derivation ergodic capacity is can be represented by following formula

$$C = (\sqrt{\eta_1\eta_2(1-|r|^2)\rho_T^2 + (\eta_1+\eta_2)\rho_T + 1}$$

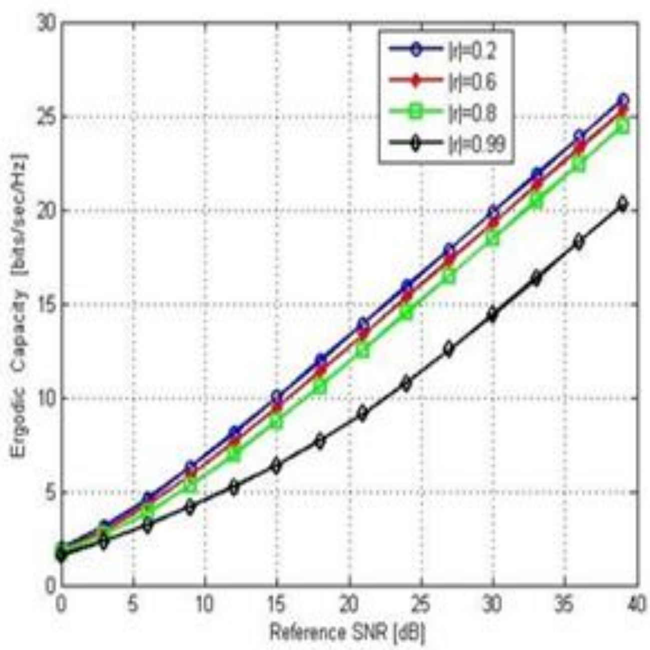

Fig.1 Ergodic capacity versus reference SNR with respect to changes in antenna correlation r $(\eta_1 = \eta_2 = 1)$.

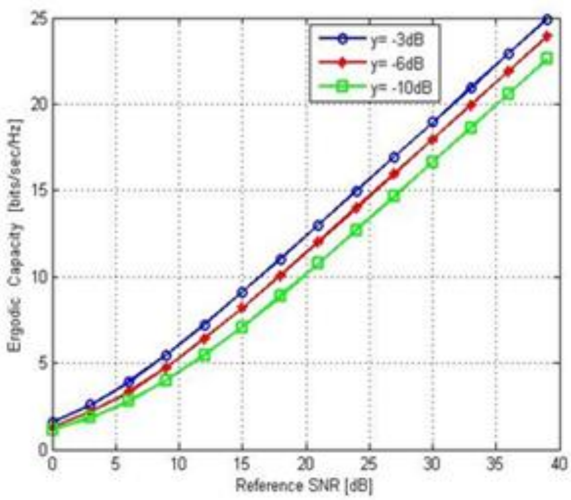

Fig.2 Ergodic capacity versus reference SNR with respect to changes in antenna efficiency imbalance $(r = 0 \text{ and } \eta_1 = 1, \eta_2 = \gamma \text{ in } \mathbf{\Lambda})$.

V. CONCLUSION

In this paper, ergodic capacity is proposed as a simple and intuitive metric for evaluating the effectiveness of MIMO terminals. This is operating in SM mode.

Impact of correlation and efficiency imbalance on ergodic capacity is illustrated by plotting graph which is helpful for an antenna engineer whose goal is to achieve the optimum antenna system design.

REFERENCES

[1] Ruiyuan Tian, Buon Kiong Lau and Zhinong Ying, “Multiplexing Efficiency of MIMO Antennas”, IEEE Antennas Wireless Propag. Lett., vol. 10, pp.183–186, 2011.

[2] B. K. Lau, “Multiple antenna terminals,” in MIMO: From Theory to Implementation, C. Oestges, A. Sibille, and A. Zanella, Eds. San Diego, CA: Academic, 2011, pp. 267–298.

[3] G. Foschini and M. Gans, “On limits of wireless communications in a fading environment when using multiple antennas,” Wireless Pers. Commun., vol. 6, no. 3, pp. 311–335, Mar. 1998.

[4] Mingxin Tan, Jigjing Chen,“Comparison and Analysis of MIMO Channel Capacity”, IEEE, pp. 299-301, , 2007.

[5] J. H. Choi and S.-O. Park, “Exact evaluation of channel capacity difference of MIMO handset antenna arrays,” IEEE Antennas Wireless Propag. Lett., vol. 9, pp. 219–222, 2010.

[6] Mingxin Tan, Jigjing Chen,“Comparison and Analysis of MIMO Channel Capacity”, IEEE, pp. 299-301, , 2007

[7] A. Paulraj, R. Nabar, and D. Gore, Introduction to Space-TimeWireless Communications. Cambridge, U.K.: Cambridge Univ. Press, 2003.

[8] Y. Fei, Y. Fan, B. K. Lau, and J. S. Thompson, “Optimal single-port matching impedance for capacity maximization in compact MIMO arrays,” IEEE Trans. Antennas Propag., vol. 56, no. 11, pp. 3566–3575, Nov. 2008.