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Stochastic Filters in Wireless Communication

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Abstract- **Stochastic filtering has become a very important tool in modern wireless communication. Adaptive based filters like Wiener Filter, Kalman Filter, Extended Kalman Filter and Unscented Kalman are being used for performance improvement of wireless communication. Keeping the above fact in mind we had tried to present a complete review on stochastic filtering, especially Wiener Filter and Kalman Filter has been analyzed in detail. In both filters, there is great interest of obtaining optimum estimation of stochastic process in order to allow their use in real time telecommunication application.**

***Keywords*-Stochastic process, Stochastic filtering, Kalman Filter, Weiner Filter, Estimation.**

I. INTRODUCTION

Stochastic filtering theory has been first established in the early 1940s because of the pioneering work by Norbert Wiener [1], [2] and Andrey N. Kolmogorov [3], [4], and it culminated in 1960 for the publication of classic Kalman filter (KF) [5] and Kalman-Bucy filter in 1961 [6], though many credits should be also given to some earlier work by Bode and Shannon [7], Zadeh and Ragazzini [8], [9], Swerling [10], Levinson , and others. It seems fair to say that the stochastic filter have completely dominated the adaptive filter theory for decades in various fields.

As we observe some natural phenomenon, there are essentially two "random" or "noisy" influences. First there may be uncontrollable, seemingly random, processes that influence the phenomenon itself, then there would be errors in the measuring process. Stochastic filters are algorithms that compute estimations of the true state of the phenomenon, given "noisy" observation[11].

Mobile communications and wireless network have experienced extensive growth and success in the recent years. However, the radio channels in mobile radio systems are usually not amiable as the wired one. Unlike wired channels that are stationary and predictable, wireless channels are extremely random and time-variant[12]. The random behavior of all wireless channel can be viewed as a stochastic process. Filtering of stochastic processes has attracted a lot of attention. The goal of stochastic filters is to determine the best estimate for the state of a stochastic process from partial observations.

Stochastic filters like Weiner and Kalman filter have wide area of applications[13]. Wiener filter has been extensively used in number of applications. For estimation of channel [11][13-16] in various system, for designing a channel equalizer[17] which remove the distortion caused from transmitted signal without requiring any specific model or state space information. Optimal Wiener filter is considered as one of the most fundamental noise reduction approaches, which has been delineated in different forms and adopted in various applications [18][19] [20].

In wireless cellular communications, accurate local mean (shadow) power estimation performed at a mobile station is important for use in power control, handoff, and adaptive transmission. A scalar Kalman-filter can be efficiently used for local mean power estimation[21] with only slightly increased computational complexity. One of the challenges in the design of wireless communication systems is accurate estimation of the channel characteristics. Kalman filter proved to be very efficient tool for channel estimation[22-31]. It is also used for interfering channels estimation and calculation of interference correlation matrix. This correlation matrix estimate is exploited in ZF or MMSE based interference cancellation scheme for interference mitigation[32-35] in next generation of the WiMAX systems[36].

II. CLASSIFICATION OF STOCHASTIC FILTERS

In past few decades a number of stochastic filters have came into existence like Weiner filter, Kalman filter[37], Extended Kalman filter, Unscented Kalman, Ensemble Kalman etc. Among them Weiner filter and Kalman filer are widely used in wireless communication. In this paper we will present complete review of two optimum linear filter, Weiner filter and Kalman Filter considering there widespread role in wireless communication.

A. Wiener Filter

The **Wiener filter** was introduced by **Norbert Wiener** in the 1940's and published in 1949 in signal processing. A major contribution in it was the use of a statistical model for the estimated signal. Wiener filters are basically a class of optimum linear filters which involve linear estimation of a desired signal sequence from another related sequence[38]. In the statistical approach for the solution of the linear filtering problem, we assume the availability of certain statistical parameters (e.g. mean

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and correlation functions) of the useful signal and unwanted additive noise. The main goal of the Wiener filter is to filter out noise that has corrupted a signal. The problem of designing a linear filter with the noisy data as input and the requirement of mitigating the effect of the noise at the filter output according to some statistical criterion. An appropriate approach to this filter-optimization problem is MMSE (minimize the mean-square value of the error signal that is defined as the difference between some desired response and the actual filter output). For stationary inputs case, the resulting solution is known as the Wiener filter.

A limitation of Wiener filter is that it is inadequate for dealing with situations in which the signal and/or noise is non-stationary[39]. In such situations, the optimum filter has to be assumed a time-varying form.

***1) Mathematical model*:**

- Assumption:
  Signal and (additive) noise are stationary linear stochastic processes with known spectral characteristics or known autocorrelation and cross-correlation.
- Requirement:
  We want to find the linear MMSE estimate of s_k based on measurement y_k. There are three versions of this problem: causal , anti-causal and FIR filter. In this paper we consider the *FIR* case for simplicity.

The whole process can be indicated as:

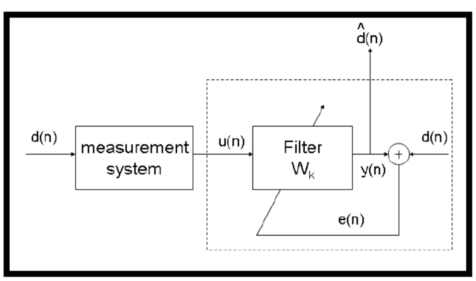

Fig.1

A linear discrete-time filter $W(z)$ for estimating a desired signal $d(n)$ based on an excitation $u(n)$. We assume that both $u(n)$ and $d(n)$ are random processes (discrete-time random signals). The filter output is $y(n)$ and $e(n)$ is the estimation error. In Weiner filter, the performance function is chosen to be

$$J(\underline{w}) = E[|e(n)|^2]$$

This is also called "mean-square error criterion"

Let $$\overline{W} = [W_0, W_1, \dots, W_{N-1}]^T$$

$$\overline{U}(n) = [u_0, u_1, \dots, u_{n-N+1}]^T$$

The output is

$$y(n) = \sum_{i=0}^{N-1} W_i u(n-i) = \overline{W}^T \overline{U}(n) = \overline{U}^T(n)\overline{W} \dots\dots\dots\dots\dots [1]$$

$$e(n) = d(n) - y(n) = d(n) - \overline{U}^T(n)\overline{W} \dots\dots\dots\dots\dots [2]$$

The performance function is then given

$$J(\underline{w}) = E[|e(n)|^2]$$

$$= E\left[\left(d(n) - \overline{W}^T\overline{U}(n)\right)\left(d(n) - \overline{U}^T(n)\overline{W}\right)\right]$$

$$= E[d^2(n)] - \overline{W}^T E[\overline{U}(n)d(n)] - E\left[\overline{U}^T(n)d(n)\right]\overline{W} + \overline{W}^T E\left[\overline{U}(n)\overline{U}^T(n)\right]\overline{W} \dots [3]$$

Now , the Nx1 cross-correlation vector is defined as:

$$\overline{P} \equiv E\left[\overline{U}(n)\, d(n)\right] = [p_0, p_1, \dots, p_{N-1}]^T$$

and the NxN autocorrelation matrix

$$R \equiv E\left[\overline{U}(n)\overline{U}^T(n)\right]$$

$$= \begin{bmatrix} r_{0,0} & r_{0,1} & \cdots & r_{0,N-1} \\ r_{1,0} & r_{1,1} & & r_{1,N-1} \\ \vdots & \vdots & & \vdots \\ r_{N-1,0} & r_{00} & \cdots & r_{N-1,N-1} \end{bmatrix}$$

$$E\left[d(n)\overline{U}^T(n)\right] = \overline{P}^T$$

$$\overline{W}^T\overline{P} = \overline{P}^T\overline{W}$$

Thus we obtain

$$J(\underline{w}) = E[d^2(n)] - 2\overline{W}^T\overline{P} + \overline{W}^T R\overline{W} \dots\dots\dots\dots\dots [4]$$

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Equation 4 is a quadratic function of the tap-weight vector $\overline{W}$ with a single global minimum.

Minimization of performance function

The set of tap weights that minimizes the performance function:

$$\frac{\partial J(\underline{w})}{\partial w_i} = 0, \text{ for } i = 0, 1, \cdots, N-1$$

From Equation 4 we have:

$$J(\underline{w}) = E[d^2(n)] - 2\sum_{l=0}^{N-1} p_l w_l + \sum_{l=0}^{N-1}\sum_{m=0}^{N-1} w_l w_m r_{lm}$$

and $\sum_{l=0}^{N-1}\sum_{m=0}^{N-1} w_l w_m r_{lm}$ can be expanded as

$$\sum_{l=0}^{N-1}\sum_{m=0}^{N-1} w_l w_m r_{lm} = \sum_{l=0,l\neq i}^{N-1}\ \sum_{m=0,m\neq i}^{N-1} w_l w_m r_{lm} + w_i \sum_{m=0,l\neq i}^{N-1} w_m r_{im} + w_i^2 r_{ii}$$

$$\frac{\partial J(\underline{w})}{\partial w_i} = -2p_i + \sum_{l=0}^{N-1} W_l(r_{li} + r_{il}), \text{ for } i = 0, 1, 2, \cdots, N-1 \ldots\ldots\ldots\ldots [5]$$

By substituting $\frac{\partial J(\underline{w})}{\partial w_i} = 0$, we obtain

$$\sum_{l=0}^{N-1} W_l(r_{li} + r_{il}) = 2p_i$$

$$r_{li} = E[x(n-l)x(n-i)] = R_{xx}(i-l)$$

$$\sum_{l=0}^{N-1} r_{il} w_l = p_i$$

In matrix notation, we then obtain

$$R\overline{W}_{op} = \overline{P} \ldots\ldots\ldots\ldots\ldots [6]$$

where $\overline{W}_{op}$ is the optimum tap-weight vector.

Equation 6 is also known as the **Wiener-Hopf equation**, which has the solution

$$\overline{W}_{op} = R^{-1}\overline{P}$$

The minimum performance function is then expressed as

$$J_{min} = E[d^2(n)] - \overline{W}_o^H R \overline{W}_o$$

B)Kalman Filter

In 1960, R.E. Kalman published his famous paper giving a recursive solution to the discrete-data linear filtering problem[40][41]. The Kalman filter is a set of mathematical equations that provides a recursive means to estimate the state of a process, which minimizes the mean of the squared error. The filter is powerful in several aspects: it enables estimations of past, present, and even future states, even when the precise nature of the modeled system is unknown. Kalman filter has various advantages from others filter. The highlighting feature of Kalman filter is that its mathematical formulation is described in terms of state space concept[42][43]. Another advantage is that the solution is computed recursively, applying without modification to the stationary as well as non-stationary environments. The main property of Kalman filter is that it is a minimum mean square (variance) estimator of the state of the linear dynamical system, which follows from a **stochastic** state space model[44].

1) Mathematical Model: We assume that we observe a process z_t at discrete instant of time $t = t_0, t_1, \ldots$, and that $z.$ follows

$z_t = H_t x_t + v_t$ (Observation equation)

where H_t is an observed design matrix, $v.$ are iid vectors of normally distributed noise with covariance, R and mean 0. It is determined by the state variable, $x.$, which we assume follows:

$\boldsymbol{x_t = Ax_{t-1} + w_{t-1}}$ (State equation)

where A is a design matrix, and iid normally distributed vectors, $w.$, with mean 0 and covariance Q. Our goal is to determine the value for $x.$ at a given time based on all information up to that time.

$$\boldsymbol{I_t = \{z_t, z_{t-1}, \ldots, z_0, x_t, x_{t-1}, \ldots, x_0\}}$$

$$\hat{x}_t = E(x_t \mid I_t)$$
$$\hat{x}_t^- = E(x_t \mid I_{t-1}).$$

A posteriori and a priori errors given by:

$$e_t = x_t - x_t$$
$$e_t^- = x_t - x_t^-$$

with corresponding error covariance matrices P_t and $\boldsymbol{P^-_t}$ .The main goal of the Kalman filter is to determine a posteriori state estimate which minimizes the a posteriori error covariance. We assume that a posteriori state is determined from the difference of the most current a priori state estimate and a linear combination of the most recent observation and the most recent predicted observation.

$$\hat{x}_t = \hat{x}_t^- + K_t(z_t - H_t\hat{x}_t^-)$$

K_t is the matrix solution to the optimization problem so defined, and is defined as Kalman gain. It has a closed form solution given by

$$K_t = P_t^- H_t (H_t P_t^- H' + R)^{-1}$$

Using the Kalman gain matrix, we update the a posteriori state variable. This can be viewed as a measurement update since a new observation is required to obtain this new state.

$$\begin{aligned} \hat{x}_t^- &= E(x_t \mid I_{t-1}) \\ &= E(Ax_{t-1} + w_{t-1} \mid I_{t-1}) \\ &= A\hat{x}_{t-1}, \end{aligned}$$

$$\begin{aligned} P_t^- &= E(e_t^- e_t^{-\prime} \mid I_{t-1}) \\ &= E((x_t - \hat{x}_t^-)^2 \mid I_{t-1}) \\ &= E((Ax_{t-1} + w_{t-1} - A\hat{x}_{t-1})^2 \mid I_{t-1}) \\ &= E((Ae_{t-1} + w_{t-1})^2 \mid I_{t-1}) \\ &= AP_{t-1}A' + Q. \end{aligned}$$

$$\begin{aligned} P_t &= P_t^- - KHP_t^- - P_t^- H'K'P_t^- H'K' \\ &= (I - KH)P_t^-. \end{aligned}$$

Putting all of this together, we have the Kalman Fillter algorithm:

$$\begin{array}{lll} i & \hat{x}_t^- = & A\hat{x}_{t-1} \\ ii & P_t^- = & AP_{t-1}A' + Q \\ iii & K_t = & P_t^- H_t'(H_t P_t^- H_t' + R)^{-1} \\ iv & \hat{x}_t = & \hat{x}_t^- + K_t(z_t - H_t\hat{x}_t^-) \\ v & P_t = & (I - K_t H_t)P_t^-. \end{array}$$

Where equation i and *ii* are the a priori, time update equations, and *iii and v* are the a posteriori, measurement update equations. The filter estimates the state process and then obtains feedback in the form of observations.

III. CONCLUSION

We have presented current investigation on application of stochastic filter in the wide field of communication. In particular role of Kalman Filter and Wiener Filter in wireless communication is addressed. Although this report is not containing any innovation but may be highly helpful for the researcher's engaged in either academic or industrial R&D in the field of wireless communication.

V. REFERENCES

[1]. N. Wiener and E. Hopf, "On a class of singular integral equations," in *Proc. Prussian Acad. Math. – Phys. Ser.*, p. 696, 1931.

[2]. N. Wiener, Extrapolation, Interpolation and smoothing of Time Series, with Engineering Applications, New York: Wiley,1949. Originally appears in 1942 as a classified National Defense Research Council Report.

[3]. A. N. Kolmogorov, "Stationary sequences in Hilbert spaces," *Bull. Math. Univ. Moscow* (in Russian), vol. 2, no. 6, p. 40, 1941.

[4]. "Interpolation and extrapolation of stationary random sequences," *Izv. Akad. Nauk USSR, Ser. Math.*, vol. 5, no. 5, pp. 3–14, 1941.

[5]. R. E. Kalman and R. S. Bucy, "New results in linear filtering and prediction theory," *Trans. ASME, Ser. D, J. Basic Eng.*, vol. 83, pp. 95– 107, 1961.

[6]. L. A. Zadeh and J. R. Ragazzini, "An extension of Wiener's theory of prediction," *J. Appl. Phys.*, vol. 21, pp. 644–655, 1950.

[7]. L. A. Zadeh, "Optimum nonlinear filters," *J. Appl. Phys.*, vol. 24, pp. 396–404, 1953.

[8]. P. Swerling, "A proposed stagewise differential correction procedure for satellite tracking and prediction," Tech. Rep. P-1292, Rand Corporation, 1958. N. Levinson, "The Wiener rms (root-mean-square) error criterion in filter design and prediction," *J. Math. Phys.*, vol. 25, pp. 261–278, Jan. 1947.

[9]. Ivan Periˇsa, Jochem Egle, and J¨urgen Lindner," Channel Estimation with Pilot-Symbols over WSSUS Channels",C. D. Charalambous, R. J. C. Bultitude, X. Li,and J. Zhan,"Modeling Wireless Fading Channels via Stochastic Differential Equations: Identification and Estimation Based on Noisy Measurements" *IEEE 2008*

[10]. Zhe Chen,"Bayesian Filtering: From Kalman Filters To Particle Filters, And Beyond" Manuscript

[11]. María Fernández-Alcalá, Jesús Navarro- Moreno," A Unified Approach to Linear Estimation Problems for Nonstationary Processes",*IEEE 2005*

[12]. Ivan Periˇsa, Jochem Egle, and J¨urgen Lindner,"Channel Estimation with Pilot-Symbols over WSSUS Channels".

[13]. *Dieter Schafhuber; Gerald Matz, and Franz Hlawatsch* Adaptive Wiener Filters For Time- Varying Channel Estimation In Wireless Ofdm Systems" Ieee Icassp-03, Hong Kong, April 2003, vol. IV, pp. 688–691

[14]. Alan Bain, Dan Crisan, "Fundamentals of Stochastic Filtering"Springer.

[15]. Hani Mehrpouyan"Channel Equalizer Design Based on Wiener Filter and Least Mean Square Algorithms"

[16]. Jingdong Chen*,"* New Insights Into the Noise Reduction Wiener Filter" *IEEE transactions on audio, speech, and language processing, vol. 14, no. 4, july 2006.*

[17]. Bram Cornelis , Marc Moonen, Jan Wouters," Performance analysis of multichannel Wiener filter based noise reduction in hearing aids under second order statistics estimation errors" *IEEE 2011.*

[18]. J. Chen, Y. Huang, and J. Benesty, "Filtering techniques for noise reduction and speech enhancement," in *Adaptive Signal Processing: Applications to Real-World Problems*, J. Benesty and Y. Huang, Eds. Berlin, Germany: Springer, 2003, pp. 129–154.

[19]. Tao Jiang, Nicholas D. Sidiropoulos,*"* Kalman Filtering for Power Estimation in Mobile Communication"*IEEE transactions on wireless communications,vol.2,no.1,January 2003 151.*

[20]. Kin K. Leung, Jack H. Winters and Leonard J.Cimini, Jr.,"Interference Estimation With Noisy Measurements in Broadband Wireless Packet Networks".

[21]. K. Sam Shanmugan, "Channel Estimation for 3G Wideband CDMA Systems Using the Kalman Filtering Algorithm"*IEEE 2000.*

[22]. Ms. Ruchi Dahiya1, Mrs. Meenu Manchanda "Measuring of channel cofficient using "KALMAN filter" *IJCSMS, Vol. 11, Issue 01, May 2011.*

[23]. Dieter Schafhubes Gerald Matz, and Franz Hlawatsch,"Kalman Tracking Of Time-Varying Channels In Wireless Mimo-Ofdm Systems"*Nov. 2003, pp. 1261–1265 IEEE 2003.*

[24]. *Dieter Schafhuber; Gerald Matz, and Franz Hlawatsch* Adaptive wiener filters for time- varying channel estimation in wireless ofdm systems" *Proc. IEEE ICASSP-03, Hong Kong, April 2003, vol. IV, pp. 688–691*

[25]. *"A Novel Channel Estimation and Tracking Method for Wireless OFDM Systems Based on Pilots and Kalman Filtering", IEEE Transactions on Consumer Electron+, Vol. 49, No. 2, MAY 2003.*

[26]. Ki-Young Han, Sang-Wook Lee, Jun-Seok Lim,"Channel Estimation for OFDM with Fast Fading Channels by Modified Kalman Filter", *IEEE Transactions 2004.*

[27]. *Wei Chen* and *Ruifeng zhang,* "Kalman-f\Filter channel estimator for ofdm systems in time and frequency selective fading environment",IEEE 2004.

[28]. M. R. Raghavendra S. Bhashyam K. Giridhar, "Parametric Channel Estimation in Reuse-1 OFDM Systems" IEEE 2007.

[29]. Fei Wang Tiejun Lv, "An Improved Kalman Filter Algorithm for UWB Channel Estimation" .

[30]. K.Krocker, "Rapid Interference suppression Using Kalman Filter Approach"1980**.**

[31]. Ching-Sheng Ni, " Co-Channel Interference suppression for Coded OFDM Systems over Frequency-Selective Slowly Fading Channels" *2004 IE*

[32]. Brian W. Kozminchuk, Asrar U. H. Sheikh,"A Kalman Filter-Based Architecture for Interference Excision"*IEEE 1995*

[33]. Nari TANABE, Toshihiro FURUKAWA, and Shigeo TSUJII ,"Fast Noise Suppression Algorithm with Kalman Filter Theory Nari TANABE",Toshihiro FURUKAWA, and Shigeo TSUJII,*2008*

[34]. Pudeyev, A. Maltsev, A. Rubtsov , S. Tiraspolsky," Selective interference cancellation using Kalman filtering"*IEEE 2006.*

[35]. Sorenson,H.*Kalman Filtering: Theory and Application*.Los Alamitos, CA: IEEE Press

[36]. Chih-Chun Feng, Chong-Yung Chi, "Design of Wiener filters using a cumulant based MSE criterion"*ELSEIVER 1996*.

[37]. S. Haykin, *Adaptive Filter Theory 3rd Edition*, Upper Saddle River,NJ: Prentice-Hall 1996

[38]. Grewal, M. and A. Andrews. "*Kalman Filtering Theory and Practice"*. Englewood Cliffs,NJ:Prentice-Hall, 1993

[39]. Gelb, A. *Applied Optimal Estimation*. Cambridge, MA: MIT Press, 1974.

[40]. WALTER T. HIGGINS,"A Comparison of Complementary and Kalman Filtering"*IEEE 1975*

[41]. Anderson, B. and J. Moore. "*Optimal Filtering"*. glewood Cliffs, NJ: Prentice-Hall, 1979.

[42]. Greg Welch ,Gary Bishop," An Introduction to the Kalman Filter"*July 24, 2006.*

[43]. Hye Mi Park and Jae Hong Lee, Shillim- Dong," Estimation of Time-Variant Channels for OFDM Systems Using Kalman and Wiener Filters"*Gwanak-Gu, Seoul 151-242.*