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Channel Estimation Using LS and MMSE Estimators

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***Abstract*— Orthogonal Frequency Division Multiplexing (OFDM) has been recently applied widely in wireless communication systems, due to its high data rate, transmission capability with high bandwidth, efficiency and its robustness to multipath delay .Channel estimation is an essential problem in OFDM system. The estimation of channel at pilot frequencies is based on Least Square, Minimum mean square error channel estimation algorithm. This paper focuses on comparison of the performances of channel estimation algorithms in terms of bit error rate, Symbol error rate and Mean square error .The modulation technique used is BPSK.**

Keywords — *MIMO, Channel estimation, Orthogonal Frequency Division Multiplexing (OFDM), bit error rate (BER), symbol error rate (SER), least square (LS), minimum mean square error (MMSE)*

I. Introduction

OFDM is a multicarrier modulation technique that has developed into a popular scheme for wideband digital communication, because of its ability to cope with severe channel conditions without complex equalization filters e.g. attenuation of high frequencies in a long copper wire, narrowband interference and frequency-selective fading due to multipath. Channel equalization is simplified because OFDM may be viewed as using many slowly modulated narrow band signals rather than one rapidly modulated wideband signal. The low symbol rate makes the use of a guard interval between symbols possible affordable, making it to handle time-spreading and eliminate ISI and ICI.

An important factor in the transmission of data is the estimation of channel which is essential before the demodulation of OFDM signals since the channel suffers from frequency selective fading and time varying factors for a particular mobile communication system. The estimation channel is mostly done by inserting pilot symbols into all of the subcarriers of an OFDM symbol or inserting pilot symbols into some of the sub-carriers of each OFDM symbol. The first method is called as the pilot based block type channel estimation. And it has been discussed for a slow fading channel. The second method is the comb-type based channel estimation in which pilot symbols are transmitted on some of the sub carriers of each OFDM symbol [4] [7]. This paper discusses the estimation of the channel using pilot arrangement which is based on Least Square (LS) Estimator and Minimum Mean-Square Error (MMSE) Estimator.

This paper aims to compare the performance of the pilot based type channel estimation by using Binary Phase Shift Keying (BPSK) modulation scheme. In Section II, the basic system model of OFDM is discussed .In Section III, the estimation of channel is performed, based on pilot arrangement. In Section IV, the simulation parameters and results are discussed. Section V concludes the findings.

II. System Description For OFDM

OFDM is an attractive modulation scheme used in broadband wireless systems that encounter large delay spreads. OFDM avoids temporal equalization altogether, using a cyclic prefix technique with a small penalty in channel capacity. Where Line of-Sight (LoS) cannot be achieved, there is likely to be significant multipath dispersion, which could limit the maximum data rate. Technologies like OFDM are probably best placed to overcome these, allowing nearly arbitrary data rates on dispersive channels. Each subcarrier can be modulated independently. The spectra of the subcarriers overlap, but the subcarrier signals are mutually orthogonal as shown in figure 1.

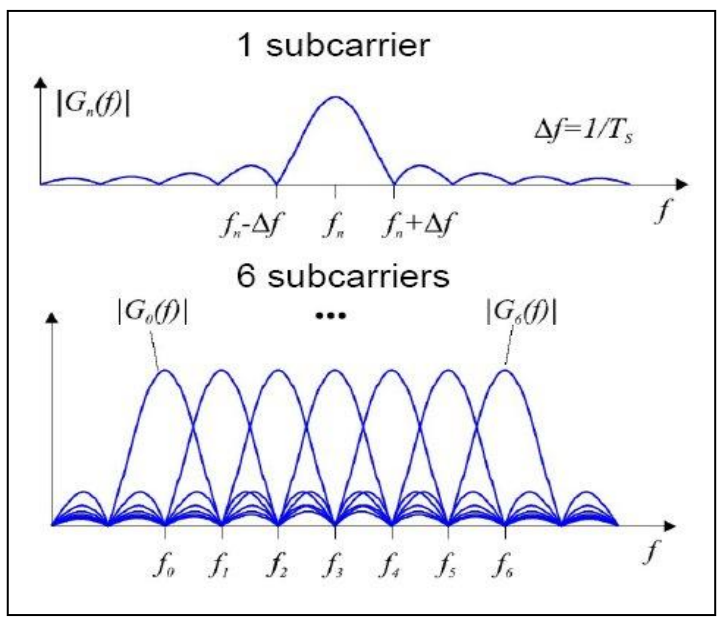

Fig 1.OFDM Subcarriers in Frequency Domain

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- OFDM Advantages

In general, OFDM systems have the following advantages: (i) efficient use of spectrum; (ii) resistant to frequency selective fading; (iii) Eliminates ISI (Inter-Symbol Interference) and ICI (Inter-Carrier Interference); (iv) can recover lost symbols due to the frequency selectivity of channels; (v) channel equalization; (vi) computationally efficient.

- OFDM Disadvantages

OFDM systems have the following disadvantages: (i) High Synchronism accuracy; (ii) Multipath propagation must be avoided in other orthogonality not be affected, and (iii) Large

Peak-to-mean power ratio due to the superposition of all subcarrier signals, this can become a distortion problem (Crest Factor).

The basic OFDM system block diagram under the assumption of frequency domain equalization is shown in figure 2.

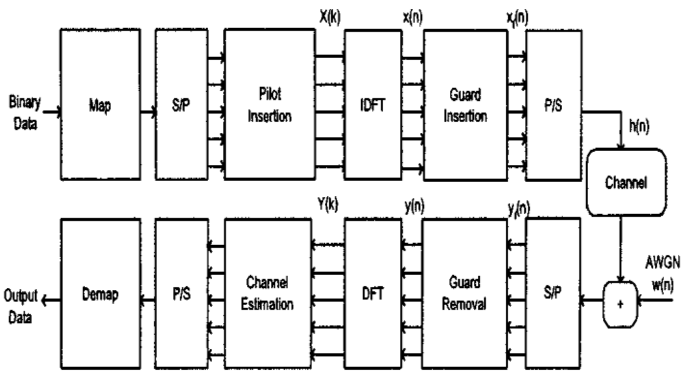

Fig 2. Baseband OFDM system.

The binary information is being generated from uniformly distributed random integers with equal probability of either 0 or 1 given as [9]:

$d_k = [d_0, d_1, d_2, \ldots\ldots d_{N-1}]$ k

$= 0, \ldots. N-1$ (1)

Higher order modulation schemes like 32 or 64 QAM are more sensitive to ISI and ICI than simple schemes like BPSK.This factor must be taken into account while deciding on the guard time.Mapper map's according to the modulation in the block of constellation mapper. d_k is converted from serial bit stream to parallel and the BPSK symbols are then superimposed on orthogonal subcarriers using IDFT given as:

$$x(k) = \sum_{n=0}^{N-1} S(n) \sin\left(\frac{2\pi kn}{N}\right) - j \sum_{n=0}^{N-1} S(n) \cos\left(\frac{2\pi kn}{N}\right) \quad (2)$$

where, *S(n)* is the BPSK/QPSK symbols and N is the length IDFT. After the IFFT block, cyclic prefix of length D, which is considered to be greater than the impulse response of the channel, it is used to combat inter-symbol interference and inter-carrier interference (ICI). It is given as:

$$x(k) = [x_{cp}(k) \;\; x(k)] \quad (3)$$

The OFDM signal is the constructed by applying the symbol along with CP to parallel to serial converter. It is then transmitted on channel given as:

$$y(k) = x(k) * h(l) + n(k) \quad (4)$$

Where, h (l) is the channel impulse response. The length of channel should be less than the cyclic prefix. For OFDM system, noise is generated in terms of symbols, so it is given as:

$$n(k) = 10^{\frac{-Es}{20N0}} * AWGN \quad (5)$$

where $\frac{Es}{N0}$ is symbol to error ratio (SER) given as

$$\left(\frac{Es}{N0}\right)_{db} = \left(\frac{N}{Ncp+N}\right)_{db} + \left(\frac{Nst}{N}\right)_{db} + \left(\frac{Es}{N0}\right)_{db} \quad (6)$$

Here, N_{cp} represents the length of cyclic prefix, is the no. of used subcarriers and is the length of FFT or no. of sub-carriers. Since the OFDM signal has overhead in terms of CP, so to compensate for it, we have to scale it so that resultant OFDM signal that is Received is given as:

$$r(k) = \sqrt{\frac{Ncp}{N}} \times y(k) \quad (7)$$

At the receiver the reverse steps are involved and since the OFDM symbols were circularly convolved with channel IR, so after FFT at the receiver the received data is equalized by using the frequency domain equalizer and the equation given as

$$X(k) = \frac{Y(k)}{H(k)} \quad (8)$$

Where, H (k) is the response of the channel in frequency domain. The frequency domain equalization is useful for equalizing the symbols that were faded as a result of experiencing multipath.

III. System Description For Channel Estimation

In block-type pilot based channel estimation, each subcarrier in an OFDM symbol is used in such a way that all sub-carriers are used as pilots. The estimation of the channel is then done using Least Square Estimator and Minimum Mean Square Error Estimator.

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$$y=DFT_N\,(IDFT\,(X) * \frac{h}{\sqrt{N}} + \hat{w}) \qquad (9)$$

Where,

$x = [x_0\ \ x_1 \ldots\ldots x_{N-1}]^T$

$y = [y_0\ \ y_1 \ldots\ldots y_{N-1}]^T$

$\hat{w} = [\hat{w}_0\ \ \hat{w}_1 \ldots\ldots \hat{w}_{N-1}]^T$

$h = [h_0\ \ h_1 \ldots\ldots h_{N-1}]^T$

The vector $\frac{h}{\sqrt{N}}$ is the observed channel impulse response when the frequency response of g(t) is sampled and it is given as :

$$h_k = \frac{h}{\sqrt{N}} \square m\, e^{-j\frac{\pi}{N}(k+(N-1)\gamma m)} \frac{\sin(\pi\gamma m)}{\sin(\frac{\pi}{N}(\gamma m-k))} \qquad (10)$$

Where, m is the length of taps, N is the no of sub carriers, and γ is the value of the tap.If inter symbol interference is eliminated by the cyclic prefix, then the system can be modeled using the equation given as,

$$y_k = H_k X_{k} + w_k \qquad k=0,\ldots N-1 \qquad (11)$$

Where H_k is the Frequency response of h given by,

$H=[H_0\ \ \ H_1 \ \ldots.. \ H_{N-1}]^T = DFT_N\,(h)$

$W=[W_0\, W_1 \ \ldots.. \ W_{N-1}]^T = DFT_N\,(\hat{w})$

Now writing the (11) in Matrix form, it becomes

$$y = XFh+w \qquad (12)$$

Here,

$X=diag\{x_0\ \ x_1 \ldots\ldots x_{N-1}\}$

$y = [y_0\ \ y_1 \ldots\ldots y_{N-1}]^T$

$\hat{w} = [\hat{w}_0\ \ \hat{w}_1 \ldots\ldots \hat{w}_{N-1}]^T$

$h = [h_0\ \ h_1 \ldots\ldots h_{N-1}]^T$

Parameters	Specification
FFT size	64
No of used Subcarriers	52
No. of OFDM symbols	100
Constellation	BPSK

$$F=\begin{bmatrix} W_N^{00} & \cdots & W_N^{N-1} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

F is the matrix of DFT with corresponding weights given as:

$$W_N^{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{nk}{N}}$$

If the channel vector h is Gaussian and is it not correlated with the noise of the channel w, then the frequency domain TABLE1:

TABLE1: SIMULATION PARAMETERS

MMSE

estimates of h becomes [2]:

$$\hat{H}_{MMSE} = FR_{hy}R^{-1}_{yy}y \qquad (13)$$

Where,

$R_{hy} = E\{hy^H\} = R_{hy}F^HX^H$

$R_{yy} = E\{yy^H\} = XFR_{hh}F^HX^H + \sigma_n^2 I_N$

Here R_{hy} is the cross correlation matrix between h and y, R_{yy} is the auto correlation matrix of y with itself and R_{hh} the auto correlation matrix of h the with itself. Since, σdenotes the noise variance .The factors R_{hh} and σ_n^2 are considered to be known. The LS estimate of the channel is given as

$$\hat{h}_{LS} = X^{-1}\, y \qquad (14)$$

Both estimators suffer from different drawbacks. The MMSE usually suffers from a high complexity, where LS estimator suffers from mean-square error which is high. The MMSE estimator requires to calculate an N * N Matrix which results in a high complexity when N becomes large.

IV. System Simulations And Results

This section discusses the results of the simulation that were performed based on the information and mathematics discussed in the Section II & III respectively. For the simulation of basic OFDM system, the following parameters are used as shown in Table.

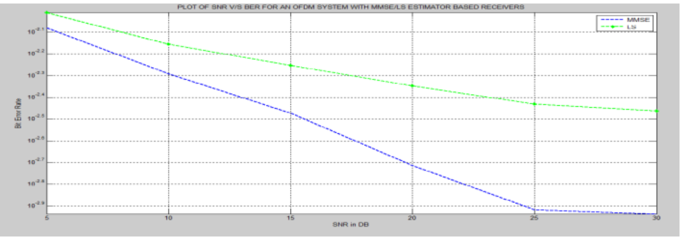

Fig 3. Plot of SNR Vs BER.

Case1:- The comparison of BER performance for the two algorithms stated above LS and MMSE is done. The simulation result in Fig. 3 shows the graph Mean Square Error (MSE) versus SNR for the LS and MMSE Estimators. For low SNR's channel noise effect is higher than the approximation effect, while it becomes dominant for large SNR's. At lower SNR value i.e 5db the BER by LS is 0.0098 and by MMSE is 0.0083 and at higher value of SNR i.e 25db the BER by LS is 0.0037 and by MMSE is 0.0012 hence the conclusion that as the value of SNR increases the difference between the BER by LS & MMSE methods also increases.

Algorithm	LS	MMSE
SNR	BER	BER
5	0.0098	0.0083
10	0.0070	0.0051
15	0.0056	0.0034
20	0.0045	0.0019
25	0.0037	0.0012

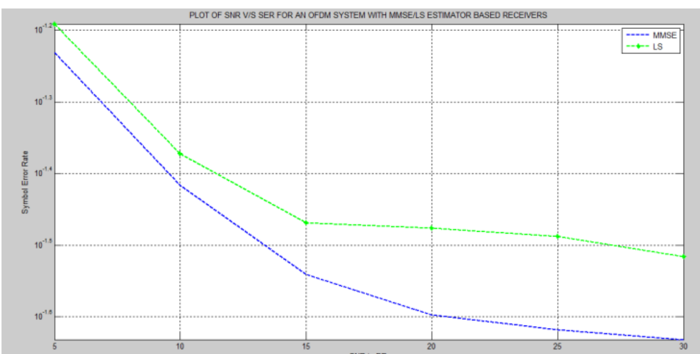

Fig 4. Plot of SNR Vs SER.

Algorithm	LS	MMSE
SNR	SER	SER
5	0.0644	0.0587
10	0.0425	0.0384
15	0.0340	0.0288
20	0.0334	0.0253
25	0.0325	0.0241

Case2:- The comparison of SER performance for the two algorithms stated above LS and MMSE is done. The simulation result in the Fig. 4 shows the graph of Symbol Error Rate (SER) versus SNR for the LS and MMSE Estimators. & The Fig. 5 shows the graph of Bit Error rate (BER) versus SNR for the LS and MMSE Estimators.

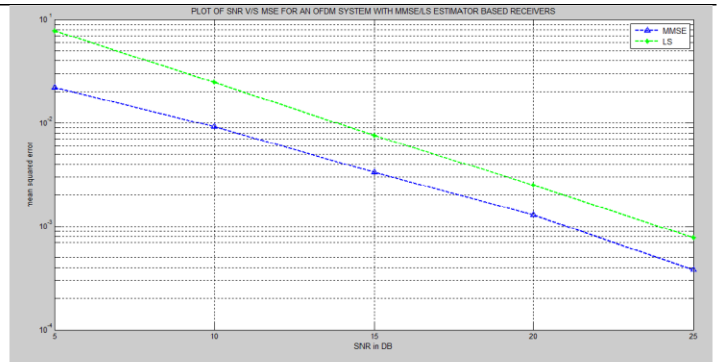

Fig 5. Plot of SNR Vs MSE.

The MMSE estimator assumes a priori knowledge of noise variance and channel covariance. So the performance of MMSE estimator is better than that of the LS estimator. Moreover, its complexity is large compared to the LS estimator. Whereas for high SNRs the LS estimator is both simple and adequate.

Algorithm	LS	MMSE
SNR	MSE	MSE
5	0.0790	0.0166
10	0.0281	0.0097
15	0.0070	0.0036
20	0.0021	0.0010
25	0.0008	0.0004

V. CONCLUSION

The paper highlights the channel estimation technique based on pilot aided block type training symbols using LS and MMSE algorithm. The Channel estimation is one of the fundamental issues of OFDM system design. The transmitted signal under goes many effects such reflection, refraction and diffraction. Also due to the mobility, the channel response can change rapidly over time. At the receiver these channel effects must be canceled to recover the original signal. The MMSE is compared with LS and the MMSE performs better than the LS where the performance metric is Bit error rate, symbol error rate, mean square.

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